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A Common Fixed Point Theorem in M-Fuzzy Metric Spaces for Integral type Inequality

Rasik M. Patel*, Ramakant Bhardwaj** and Rashmi Tiwari*

**Research Scholar of Sai Nath University, Ranchi (Jharkhand)*
S.P.B. Patel Engineering College, Mehsana
***TIT group of Institutes (TIT-Excellence) Bhopal, (Madhya Pradesh), INDIA*
****Department of Mathematics, Govt. NMV Hoshangabad, (Madhya Pradesh), INDIA*

(Corresponding author: Rasik M. Patel)
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ABSTRACT: In this paper, we mainly give a common fixed point theorem for three pairs of weakly compatible maps in M-fuzzy metric spaces for integral type inequality by introducing Common property (E) for two pairs of mappings.

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Key words and phrases: M-fuzzy metric space, Compatible mappings, Common fixed point theorem, Common property (E)

I. INTRODUCTION

The concept of sets was introduced by Zadeh [9] in 1965, I. Kramosil and J. Michalek [7] introduced the concept of fuzzy topological spaces induced by fuzzy metric which have very important applications in quantum particle physics. Consequently in due course of time many researchers have defined a fuzzy metric space in different ways. Researchers like A. George and P. Veeramani [2], M. Grabiec [10], P.V. Subrahmanyam [11], R. Vasuki [12] used this concept to generalize some metric fixed point results. Recently, Sedghi and Shobe [17] introduced M-fuzzy metric space which is based on D*-metric concept. Dhage [5] introduced the notion of generalized metric or D-metric spaces by S.V.R. Naidu, K.P.R. Rao and N.Srinivasa Rao [14], [15], [16] and proved several fixed point theorems in it. Also proved a Common fixed point theorem for three pairs of maps in M-Fuzzy metric spaces by K. P. R. Rao, G. Ravi Babu and V.C.C. Raju [8].

II. PRELIMINARIES

Definition 2.1 [1]: Let (X, d) be a complete metric space, c∈ (0, 1) and f: X→X be a mapping such that for each x, y X,

$$\int_0^{d(fx,fy)} \varphi(t)dt \le c\int_0^{d(x,y)} \varphi(t)dt$$

where φ: [0,+∞) →[0,+∞) is a Lebesgue integrable mapping which is summable on each compact subset of [0, +∞) , non negative, and such that for each $\varepsilon > 0$, $\int_0^{\varepsilon} \varphi(t)dt$, then f has a unique fixed point $a \in X$ such that for each $x \in X$, $\lim_{n\to\infty} f^n x = a$.

B.E.Rhoades [3], extending the result of Branciari by replacing the above condition by the following

$$\int_0^{d(fx,fy)} \varphi(t)dt \le c\int_0^{max\left\{d(x,y),d(x,fx),d(y,fy),\frac{d(x,fy)+d(y,fx)}{2}\right\}} \varphi(t)dt.$$

Definition 2.2 [4]: A binary operation *: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if it satisfies the following conditions:

(1) * is associative and commutative,
(2) * is continuous,
(3) a * 1 = a for all a ∈ [0, 1],

(4) a * b ≤ c * d whenever a ≤ c and b ≤ d for all a, b, c, d □ [0, 1],

Two typical examples of continuous t-norm are a * b = ab and a * b = min (a, b).

Definition 2.3 [17]: A 3-tuple (X, M,*) is called a M-fuzzy metric space if X is an arbitrary (Non-empty) set, * is a continuous t-norm and M is a fuzzy set on $X^3 \times (0,\infty)$satisfying the following conditions: for all x, y, z, $a \in X$ and t, s > 0,

(1) $M(x,y,t) > 0$.

(2) $M(x,y,z,t) = 1$ if and only if $x = y = z$,

(3) $M(x,y,z,t) = M(p\{x,y,z\},t)$,(symmetry) where p is a permutation function

(4) $M(x,y,a,t) * M(a,z,z,s) \leq M(x,y,z,t+s)$.

(5) $M(x,y,z,.) : (0,\infty) \to [0,1]$ is continuous.

Remark 2.1 [17]. Let (X, M,*) be a M-fuzzy metric space. Then for every t > 0 and for every x, y □ X, we have $M(x,x,y,t) = M(x,y,y,t)$.

Definition 2.4 [**17**]. Let (X, M,*) be a M-fuzzy metric space. For t > 0, the open ball $B_M(x,r,t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by $B_M(x,r,t) = \{y \in X\colon M(x,y,y,t) > 1 - r\}$. A subset A of X is called open set if for each $x \in A$ there exist t > 0 and 0 < r < 1 such that $B_M(x,r,t) \subseteq A$.

Definition 2.5 [17]. A sequence $\{x_n\}$ in X converges to x if and only if M $(x,x,x_n,t) \to 1$ as $n \to \infty$, for each t > 0. It is called a Cauchy sequence if for each $0 < \epsilon < 1$ and t > 0, there exists $n_0 \in \mathbf{N}$ such that M $(x_n, x_n, x_m, t) > 1 - \epsilon$ for each n, m $\geq n_0$. The M-fuzzy metric space (X, M,*) is said to be complete if every Cauchy sequence is convergent.

Lemma 2.1[17]. Let (X, M,*) be a M-fuzzy metric space. Then M(x,y,z,t) is non decreasing with respect to t, for all x,y,z in X.

Lemma 2.2 [**17**]. Let (X, M,*) be a M-fuzzy metric space. Then M is Continuous function on $X^3 \times (0,\infty)$.

Definition 2.6[17]. Let f and g be two self maps of (X, M,*). Then f and g are said to satisfy property (E), if there exists a sequence $\{x_n\}$ in X such that M $(fx_n,u,u,t) \to 1$ & M $(gx_n,u,u,t) \to 1$ as $n \to \infty$ for some u in X and for every t > 0.

Liu et.al [18] defined common property (E) for two pairs of maps in a metric space. In 1998, Jungck and Rhoades [6] introduced the concept of weakly compatibility of pair of self mappings in a metric space.

Definition 2.7[17]. Let f and g be two self maps of (X, M,*).Then f and g are said to be weakly compatible if there exists u in X with $fu = gu$ implies $fgu = gfu$.

Definition 2.8[8]. Let P,Q,f and g be self mappings on M-fuzzy metric space (X, M,*).We say that the pairs (P,f) and (Q,g) satisfy common property (E) if there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that M$(Px_n,u,u,t) \to 1$, M$(fx_n,u,u,t) \to 1$,M$(Qy_n,u,u,t) \to 1$ and M$(gy_n,u,u,t) \to 1$ as $n \to \infty$ for some u in X and for every t > 0.

Example 2.1[8]. Let X = **R** & $M(x,y,z,t) = \frac{t}{t+|x-y|+|y-z|+|x-z|}$ for all t > 0 & $x,y,z \in X$. Let $P,Q,f,g\colon X \to X$ be defined by $Px = 2x+1, fx = x+2, Qx = 2x+5$ and $gx = 2-x$. Consider the sequences $\{x_n\} = \{1+\frac{1}{n}\}$and $\{y_n\} = \{-1+\frac{1}{n}\}$.

Then M $(Px_n,3,3,t) \to 1$, M $(fx_n,3,3,t) \to 1$, M $(Qx_n,3,3,t) \to 1$ and M $(gx_n,3,3,t) \to 1$ as $n \to \infty$ for every t > 0.Thus the pairs (P,f) and (Q,g) satisfy common property (E).

3. Main Results

Theorem 3.1 Let P,Q,R,f,g and h be self mappings of a M-fuzzy metric space (X, M, *) satisfying

(3.1.1) $P(X) \subseteq g(X)$,$Q(X) \subseteq h(X), R(X) \subseteq f(X)$ and $f(X)$ or $g(X)$ or $h(X)$ is a closed subspace of X,

(3.1.2) the pairs $(P,f),(Q,g)$ and (R,h) are weakly compatible,

(3.1.3) any two pairs from $(P,f),(Q,g)$ and (R,h) satisfy common property (E) and

(3.1.4)

$$\int_0^{M(Px,Qy,Rz,\emptyset(t))} \xi(t)dt \geq \int_0^{\left\{\begin{matrix} \varphi\left(min\begin{pmatrix} M(fx,gy,hz,t),M(fx,Px,Qy,t), \\ M(gy,Qy,Rz,t),M(hz,Rz,Px,t), \\ M(fx,gy,Rz,t),M(Px,gy,hz,t)\end{pmatrix}\right) \\ + \\ \delta\left(min\left(\frac{M(fx,Qy,hz,t)}{M(fx,gy,hz,t)},\frac{M(Px,Qy,hz,t)}{M(Px,gy,hz,t)}\right)\right)\end{matrix}\right\}} \xi(t)dt$$

□$x,y,z \in X, \forall\, t > 0$, and $\varphi,\delta,\emptyset$: (0,∞) → (0,∞) is such that □$(t) < t$, $\varphi(t) < t, \delta(t) < t$.

Then P, Q, R, f, g and h have a unique common fixed point in X.

Proof.

Suppose the pairs (P, f) and (Q, g) satisfy common property (E). Then there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} fx_n = \lim_{n\to\infty} Qy_n = \lim_{n\to\infty} gy_n = \alpha$$

For some $\alpha \in X$.

Since $Q(X) \subseteq h(X)$, there exists a sequence $\{z_n\}$ in X such that $Qy_n = h\, z_n$, ∎ n. Hence $\lim_{n\to\infty} hz_n = \alpha$. $let \lim_{n\to\infty} Rz_n = \gamma$.

Now from (3.1.4), we have

$$\int_0^{M(Px_n,Qy_n,Rz_n,\emptyset(t))} \xi(t)dt \geq \int_0^{\left\{\begin{matrix} \varphi\left(min\begin{pmatrix} M(fx_n,gy_n,hz_n,t),M(fx_n,Px_n,Qy_n,t), \\ M(gy_n,Qy_n,Rz_n,t),M(hz_n,Rz_n,Px_n,t), \\ M(fx_n,gy_n,Rz_n,t),M(Px_n,gy_n,hz_n,t) \end{pmatrix}\right) \\ + \\ \delta\left(min\left(\frac{M(fx_n,Qy_n,hz_n,t)}{M(fx_n,gy_n,hz_n,t)},\frac{M(Px_n,Qy_n,hz_n,t)}{M(Px_n,gy_n,hz_n,t)}\right)\right) \end{matrix}\right\}} \xi(t)dt$$

Letting $n \to \infty$, we get

$$\int_0^{M(\alpha,\alpha,\gamma,\emptyset(t))} \xi(t)dt \geq \int_0^{\left\{\begin{matrix} \varphi\left(min\begin{pmatrix} M(\alpha,\alpha,\alpha,t),M(\alpha,\alpha,\alpha,t), \\ M(\alpha,\alpha,\gamma,t),M(\alpha,\gamma,\alpha,t), \\ M(\alpha,\alpha,\gamma,t),M(\alpha,\alpha,\alpha,t) \end{pmatrix}\right) \\ + \\ \delta\left(min\left(\frac{M(\alpha,\alpha,\alpha,t)}{M(\alpha,\alpha,\alpha,t)},\frac{M(\alpha,\alpha,\alpha,t)}{M(\alpha,\alpha,\alpha,t)}\right)\right) \end{matrix}\right\}} \xi(t)dt$$

$$\int_0^{M(\alpha,\alpha,\gamma,\emptyset(t))} \xi(t)dt \geq \int_0^{\left\{\begin{matrix} \varphi(M(\alpha,\alpha,\gamma,t)) \\ + \\ \delta(1) \end{matrix}\right\}} \xi(t)dt \geq \int_0^{\{M(\alpha,\alpha,\gamma,t)\}} \xi(t)dt$$

$$i.e \int_0^{M(\alpha,\alpha,\gamma,\emptyset(t))} \xi(t)dt \geq \int_0^{\{M(\alpha,\alpha,\gamma,t)\}} \xi(t)dt$$

So that $\gamma = \alpha$.

$Thus \lim_{n\to\infty} Rz_n = \alpha$

Suppose $f(X)$ is a closed subspace of X. Then $\alpha = fu$ for some $u \in X$. Now in (3.1.4)

$$\int_0^{M(Pu,Qy_n,Rz_n,\emptyset(t))} \xi(t)dt \geq \int_0^{\left\{\begin{matrix} \varphi\left(min\begin{pmatrix} M(fu,gy_n,hz_n,t),M(fu,Pu,Qy_n,t), \\ M(gy_n,Qy_n,Rz_n,t),M(hz_n,Rz_n,Pu,t), \\ M(fu,gy_n,Rz_n,t),M(Pu,gy_n,hz_n,t) \end{pmatrix}\right) \\ + \\ \delta\left(min\left(\frac{M(fu,Qy_n,hz_n,t)}{M(fu,gy_n,hz_n,t)},\frac{M(Pu,Qy_n,hz_n,t)}{M(Pu,gy_n,hz_n,t)}\right)\right) \end{matrix}\right\}} \xi(t)dt$$

Letting $n \to \infty$, we get

$$\int_0^{M(Pu,\alpha,\alpha,\emptyset(t))} \xi(t)dt \geq \int_0^{\left\{\begin{matrix} \varphi\left(min\begin{pmatrix} M(\alpha,\alpha,\alpha,t),M(\alpha,Pu,\alpha,t), \\ M(\alpha,\alpha,\alpha,t),M(\alpha,\alpha,Pu,t), \\ M(\alpha,\alpha,\alpha,t),M(Pu,\alpha,\alpha,t) \end{pmatrix}\right) \\ + \\ \delta\left(min\left(\frac{M(\alpha,\alpha,\alpha,t)}{M(\alpha,\alpha,\alpha,t)},\frac{M(Pu,\alpha,\alpha,t)}{M(Pu,\alpha,\alpha,t)}\right)\right) \end{matrix}\right\}} \xi(t)dt$$

$$\int_0^{M(Pu,\alpha,\alpha,\emptyset(t))} \xi(t)dt \geq \int_0^{\left\{\begin{matrix}\varphi(M(Pu,\alpha,\alpha,t)) \\ + \\ \delta(1)\end{matrix}\right\}} \xi(t)dt \geq \int_0^{\{M(Pu,\alpha,\alpha,t)\}} \xi(t)dt$$

$$i.e \int_0^{M(Pu,\alpha,\alpha,\emptyset(t))} \xi(t)dt \geq \int_0^{\{M(Pu,\alpha,\alpha,t)\}} \xi(t)dt$$

So that $Pu = \alpha$.
Since the pair (P, f) is weakly compatible and $Pu = fu = \alpha$, we have $P\alpha = f\alpha$.
Since $P(X) \subset g(X)$, there exists $v \in X$ such that $\alpha = Pu = gv$.Now we have (3.1.4)

$$\int_0^{M(Pu,Qv,Rz_n,\emptyset(t))} \xi(t)dt \geq \int_0^{\left\{\begin{matrix}\varphi\left(\min\begin{pmatrix}M(fu,gv,hz_n,t),M(fu,Pu,Qv,t),\\ M(gv,Qv,Rz_n,t),M(hz_n,Rz_n,Pu,t),\\ M(fu,gv,Rz_n,t),M(Pu,gv,hz_n,t)\end{pmatrix}\right) \\ + \\ \delta\left(\min\left(\frac{M(fu,Qv,hz_n,t)}{M(fu,gv,hz_n,t)},\frac{M(Pu,Qv,hz_n,t)}{M(Pu,gv,hz_n,t)}\right)\right)\end{matrix}\right\}} \xi(t)dt$$

Letting $n \to \infty$, we get

$$\int_0^{M(\alpha,Qv,\alpha,\emptyset(t))} \xi(t)dt \geq \int_0^{\left\{\begin{matrix}\varphi\left(\min\begin{pmatrix}M(\alpha,\alpha,\alpha,t),M(\alpha,\alpha,Qv,t),\\ M(\alpha,Qv,\alpha,t),M(\alpha,\alpha,\alpha,t),\\ M(\alpha,\alpha,\alpha,t),M(\alpha,\alpha,\alpha,t)\end{pmatrix}\right) \\ + \\ \delta\left(\min\left(\frac{M(\alpha,Qv,\alpha,t)}{M(\alpha,\alpha,\alpha,t)},\frac{M(\alpha,Qv,\alpha,t)}{M(\alpha,\alpha,\alpha,t)}\right)\right)\end{matrix}\right\}} \xi(t)dt$$

$$\int_0^{M(\alpha,Qv,\alpha,\emptyset(t))} \xi(t)dt \geq \int_0^{\left\{\begin{matrix}\varphi(M(\alpha,Qv,\alpha,t)) \\ + \\ \delta(M(\alpha,Qv,\alpha,t))\end{matrix}\right\}} \xi(t)dt \geq \int_0^{\{M(\alpha,Qv,\alpha,t)\}} \xi(t)dt$$

$$i.e \int_0^{M(\alpha,Qv,\alpha,\emptyset(t))} \xi(t)dt \geq \int_0^{\{M(\alpha,Qv,\alpha,t)\}} \xi(t)dt$$

So that $Q\,v = \alpha$.
Since the pair (Q,g) is weakly compatible and $Q\,v = gv = \alpha$, we have $Q\alpha = g\alpha$.
Since $Q(X) \subset h(X)$, there exists $w \in X$ such that $\alpha = Q\,v = hw$. Now we have (3.1.4)

$$\int_0^{M(Pu,Qv,Rw,\emptyset(t))} \xi(t)dt \geq \int_0^{\left\{\begin{matrix}\varphi\left(\min\begin{pmatrix}M(fu,gv,hw,t),M(fu,Pu,Qv,t),\\ M(gv,Qv,Rw,t),M(hw,Rw,Pu,t),\\ M(fu,gv,Rw,t),M(Pu,gv,hw,t)\end{pmatrix}\right) \\ + \\ \delta\left(\min\left(\frac{M(fu,Qv,hw,t)}{M(fu,gv,hw,t)},\frac{M(Pu,Qv,hw,t)}{M(Pu,gv,hw,t)}\right)\right)\end{matrix}\right\}} \xi(t)dt$$

$$\int_0^{M(\alpha,\alpha,Rw,\emptyset(t))} \xi(t)dt \geq \int_0^{\left\{\begin{array}{c}\varphi\left(min\begin{pmatrix}M(\alpha,\alpha,\alpha,t),M(\alpha,\alpha,\alpha,t),\\ M(\alpha,\alpha,Rw,t),M(\alpha,Rw,\alpha,t),\\ M(\alpha,\alpha,Rw,t),M(\alpha,\alpha,\alpha,t)\end{pmatrix}\right)\\ +\\ \delta\left(min\left(\frac{M(\alpha,\alpha,\alpha,t)}{M(\alpha,\alpha,\alpha,t)},\frac{M(\alpha,\alpha,\alpha,t)}{M(\alpha,\alpha,\alpha,t)}\right)\right)\end{array}\right\}} \xi(t)dt$$

$$\int_0^{M(\alpha,\alpha,Rw,\emptyset(t)} \xi(t)dt \geq \int_0^{\left\{\begin{array}{c}\varphi(M(\alpha,\alpha,Rw,t))\\ +\\ \delta(1)\end{array}\right\}} \xi(t)dt \geq \int_0^{\{M(\alpha,\alpha,Rw,t)\}} \xi(t)dt$$

$$i.e \int_0^{M(\alpha,\alpha,Rw,\emptyset(t))} \xi(t)dt \geq \int_0^{\{M(\alpha,\alpha,Rw,t)\}} \xi(t)dt$$

So that $Rw = \alpha$.
Since the pair (R, h) is weakly compatible, we have $R\alpha = h\alpha$. From (3.1.4) we have

$$\int_0^{M(P\alpha,\alpha,\alpha,\emptyset(t))} \xi(t)dt \geq \int_0^{M(P\alpha,Qv,Rw,\emptyset(t))} \xi(t)dt$$

$$\geq \int_0^{\left\{\begin{array}{c}\varphi\left(min\begin{pmatrix}M(f\alpha,gv,hw,t),M(f\alpha,P\alpha,Qv,t),\\ M(gv,Qv,Rw,t),M(hw,Rw,P\alpha,t),\\ M(f\alpha,gv,Rw,t),M(P\alpha,gv,hw,t)\end{pmatrix}\right)\\ +\\ \delta\left(min\left(\frac{M(f\alpha,Qv,hw,t)}{M(f\alpha,gv,hw,t)},\frac{M(P\alpha,Qv,hw,t)}{M(P\alpha,gv,hw,t)}\right)\right)\end{array}\right\}} \xi(t)dt$$

$$\geq \int_0^{\left\{\begin{array}{c}\varphi\left(min\begin{pmatrix}M(P\alpha,\alpha,\alpha,t),M(P\alpha,P\alpha,\alpha,t),\\ M(\alpha,\alpha,\alpha,t),M(\alpha,\alpha,P\alpha,t),\\ M(P\alpha,\alpha,\alpha,t),M(P\alpha,\alpha,\alpha,t)\end{pmatrix}\right)\\ +\\ \delta\left(min\left(\frac{M(P\alpha,\alpha,\alpha,t)}{M(P\alpha,\alpha,\alpha,t)},\frac{M(P\alpha,\alpha,\alpha,t)}{M(P\alpha,\alpha,\alpha,t)}\right)\right)\end{array}\right\}} \xi(t)dt$$

$$\int_0^{M(P\alpha,\alpha,\alpha,\emptyset(t)} \xi(t)dt \geq \int_0^{\left\{\begin{array}{c}\varphi(M(P\alpha,\alpha,\alpha,t))\\ +\\ \delta(1)\end{array}\right\}} \xi(t)dt \geq \int_0^{\{M(P\alpha,\alpha,\alpha,t)\}} \xi(t)dt$$

$$i.e \int_0^{M(P\alpha,\alpha,\alpha,\emptyset(t))} \xi(t)dt \geq \int_0^{\{M(P\alpha,\alpha,\alpha,t)\}} \xi(t)dt$$

From remark (2.1), So that $P\alpha = \alpha$ and $P\alpha = \alpha = f\alpha$. Now we have

$$\int_0^{M(\alpha,Q\alpha,\alpha,\emptyset(t))} \xi(t)dt \geq \int_0^{M(P\alpha,Q\alpha,Rw,\emptyset(t))} \xi(t)dt$$

$$\geq \int_0^{\left\{\begin{array}{c}\varphi\left(min\begin{pmatrix}M(f\alpha,g\alpha,hw,t),M(f\alpha,P\alpha,Q\alpha,t),\\ M(g\alpha,Q\alpha,Rw,t),M(hw,Rw,P\alpha,t),\\ M(f\alpha,g\alpha,Rw,t),M(P\alpha,g\alpha,hw,t)\end{pmatrix}\right)\\ +\\ \delta\left(min\left(\frac{M(f\alpha,Q\alpha,hw,t)}{M(f\alpha,g\alpha,hw,t)},\frac{M(P\alpha,Q\alpha,hw,t)}{M(P\alpha,g\alpha,hw,t)}\right)\right)\end{array}\right\}} \xi(t)dt$$

$$\geq \int_0^{\left\{\begin{array}{c}\varphi\left(min\begin{pmatrix}M(\alpha,Q\alpha,\alpha,t),M(\alpha,\alpha,Q\alpha,t),\\ M(Q\alpha,Q\alpha,\alpha,t),M(\alpha,\alpha,\alpha,t),\\ M(\alpha,Q\alpha,\alpha,t),M(\alpha,Q\alpha,\alpha,t)\end{pmatrix}\right)\\ +\\ \delta\left(min\left(\frac{M(\alpha,Q\alpha,\alpha,t)}{M(\alpha,Q\alpha,\alpha,t)},\frac{M(\alpha,Q\alpha,\alpha,t)}{M(\alpha,Q\alpha,\alpha,t)}\right)\right)\end{array}\right\}} \xi(t)dt$$

$$\int_0^{M(\alpha,Q\alpha,\alpha,\emptyset(t)} \xi(t)dt \geq \int_0^{\left\{\begin{array}{c}\varphi(M(\alpha,Q\alpha,\alpha,t))\\ +\\ \delta(1)\end{array}\right\}} \xi(t)dt \geq \int_0^{\{M(\alpha,Q\alpha,\alpha,t)\}} \xi(t)dt$$

$$i.e \int_0^{M(\alpha,Q\alpha,\alpha,\emptyset(t))} \xi(t)dt \geq \int_0^{\{M(\alpha,Q\alpha,\alpha,t)\}} \xi(t)dt$$

From remark (2.1), So that $Q\,\alpha = \alpha$ and hence $Q\,\alpha = \alpha = g\,\alpha$. Now we have

$$\int_0^{M(\alpha,\alpha,R\alpha,\emptyset(t))} \xi(t)dt \geq \int_0^{M(P\alpha,Q\alpha,R\alpha,\emptyset(t))} \xi(t)dt$$

$$\geq \int_0^{\left\{\begin{array}{c}\varphi\left(min\begin{pmatrix}M(f\alpha,g\alpha,h\alpha,t),M(f\alpha,P\alpha,Q\alpha,t),\\ M(g\alpha,Q\alpha,R\alpha,t),M(h\alpha,R\alpha,P\alpha,t),\\ M(f\alpha,g\alpha,R\alpha,t),M(P\alpha,g\alpha,h\alpha,t)\end{pmatrix}\right)\\ +\\ \delta\left(min\left(\frac{M(f\alpha,Q\alpha,h\alpha,t)}{M(f\alpha,g\alpha,h\alpha,t)},\frac{M(P\alpha,Q\alpha,h\alpha,t)}{M(P\alpha,g\alpha,h\alpha,t)}\right)\right)\end{array}\right\}} \xi(t)dt$$

$$\geq \int_0^{\left\{\begin{array}{c}\varphi\left(min\begin{pmatrix}M(\alpha,\alpha,R\alpha,t),M(\alpha,\alpha,\alpha,t),\\ M(\alpha,\alpha,R\alpha,t),M(R\alpha,R\alpha,\alpha,t),\\ M(\alpha,\alpha,R\alpha,t),M(\alpha,\alpha,R\alpha,t)\end{pmatrix}\right)\\ +\\ \delta\left(min\left(\frac{M(\alpha,\alpha,R\alpha,t)}{M(\alpha,\alpha,R\alpha,t)},\frac{M(\alpha,\alpha,R\alpha,t)}{M(\alpha,\alpha,R\alpha,t)}\right)\right)\end{array}\right\}} \xi(t)dt$$

$$\int_0^{M(\alpha,\alpha,R\alpha,\emptyset(t)} \xi(t)dt \geq \int_0^{\left\{\begin{array}{c}\varphi(M(\alpha,\alpha,R\alpha,t))\\ +\\ \delta(1)\end{array}\right\}} \xi(t)dt \geq \int_0^{\{M(\alpha,\alpha,R\alpha,t)\}} \xi(t)dt$$

$$i.e \int_0^{M(\alpha,\alpha,R\alpha,\emptyset(t))} \xi(t)dt \geq \int_0^{\{M(\alpha,\alpha,R\alpha,t)\}} \xi(t)dt$$

From remark (2.1), So that $R\,\alpha = \alpha$ and hence $R\,\alpha = \alpha = h\,\alpha$.
Thus $P\alpha = Q\alpha = R\alpha = f\alpha = g\alpha = h\alpha = \alpha$
Suppose that $\beta \neq \alpha$ is a common fixed point of *P, Q, R, f, g* and *h*.

$$\int_0^{M(\beta,\alpha,R\alpha,\emptyset(t))} \xi(t)dt \geq \int_0^{M(P\beta,Q\alpha,R\alpha,\emptyset(t))} \xi(t)dt$$

$$\geq \int_0^{\left\{\begin{array}{c}\varphi\left(min\begin{pmatrix}M(f\beta,g\alpha,h\alpha,t),M(f\beta,P\beta,Q\alpha,t),\\ M(g\alpha,Q\alpha,R\alpha,t),M(h\alpha,R\alpha,P\beta,t),\\ M(f\beta,g\alpha,R\alpha,t),M(P\beta,g\alpha,h\alpha,t)\end{pmatrix}\right)\\ +\\ \delta\left(min\left(\frac{M(f\beta,Q\alpha,h\alpha,t)}{M(f\beta,g\alpha,h\alpha,t)},\frac{M(P\beta,Q\alpha,h\alpha,t)}{M(P\beta,g\alpha,h\alpha,t)}\right)\right)\end{array}\right\}} \xi(t)dt$$

$$\geq \int_0^{\left\{\begin{array}{c}\varphi\left(min\begin{pmatrix}M(\beta,\alpha,\alpha,t),M(\beta,\beta,\alpha,t),\\ M(\alpha,\alpha,\alpha,t),M(\alpha,\alpha,\beta,t),\\ M(\beta,\alpha,\alpha,t),M(\beta,\alpha,\alpha,t)\end{pmatrix}\right)\\ +\\ \delta\left(min\left(\frac{M(\beta,\alpha,\alpha,t)}{M(\beta,\alpha,\alpha,t)},\frac{M(\beta,\alpha,\alpha,t)}{M(\beta,\alpha,\alpha,t)}\right)\right)\end{array}\right\}} \xi(t)dt$$

$$\int_0^{M(\beta,\alpha,\alpha,\emptyset(t)} \xi(t)dt \geq \int_0^{\left\{\begin{array}{c}\varphi(M(\beta,\alpha,\alpha,t))\\ +\\ \delta(1)\end{array}\right\}} \xi(t)dt \geq \int_0^{\{M(\beta,\alpha,\alpha,t)\}} \xi(t)dt$$

$$i.e \int_0^{M(\beta,\alpha,\alpha,\emptyset(t))} \xi(t)dt \geq \int_0^{\{M(\beta,\alpha,\alpha,t)\}} \xi(t)dt$$

From Remark (2.1)
So that $\beta = \alpha$. Thus α is the unique common fixed point of P, Q, R, f, g and h.

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